On the nonlinear response of a particle interacting with fermions in a 1D lattice

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By the Bethe ansatz method we study the energy dispersion of a particle interacting by a local interaction with fermions (or hard core bosons) of equal mass in a one dimensional lattice. We focus on the period of the Bloch oscillations which turns out to be related to the Fermi wavevector of the Fermi sea and in particular on how this dispersion emerges as a collective effect in the thermodynamic limit. We show by symmetry that the dispersion is temperature independent for a half-filled system. We also discuss the adiabatic coherent collective response of the particle to an applied field.

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Prototype integrable quantum many-body problems provide valuable insight into the physics of correlated electronic and magnetic systems. With the development of novel experimental systems, as quasi-one dimensional quantum magnets [1] and cold atom systems [2], it became possible to tailor make and experimentally study these systems. At the same time because of their non-generic character they exhibit unconventional dynamic and transport properties of academic and potentially technological interest.

One of the simplest models that was studied early on [3], is that of a particle interacting with spinless fermions in a one dimensional lattice, where the mass of the extra particle is the same as the mass of the fermions. In this model and its lattice version, it was shown that, despite the particle-fermion interaction, the transport of the "heavy" particle remains ballistic at all temperatures, namely its mobility diverges [4, 5]. The spinless fermion or hard core boson bath corresponds to the Tonks-Girardeau gas, a one dimensional boson model with very strong local repulsion.

More recently [6–8], motivated by cold atom physics experiments, the dispersion and period of Bloch oscillations of the heavy particle when acted by a constant field in a continuous system were also studied and shown to be related to the Fermi wavevector k_F of the fermionic sea. The first issue we study in this work is the competition between the period of oscillations imposed by the lattice in the tight binding version of the model and that imposed by the Fermi wavevector.

The lattice model corresponds to a particular sector of the 1D Hubbard model [5], a prototype integrable model by the Bethe ansatz method [9]. The analytical solution allows us to characterize the low energy spectrum and also study the adiabatic response of the particle to an applied field.

The system is described by the Hamiltonian,

$$\begin{aligned}\hat{H} &= -t_h \sum_l (e^{i\phi} d_{l+1}^\dagger d_l + h.c.) - t \sum_l (c_{l+1}^\dagger c_l + h.c.) \\ &+ U \sum_l d_l^\dagger d_l c_l^\dagger c_l,\end{aligned}$$

where $c_l(c_l^\dagger)$ are annihilation (creation) operators for N spinless fermions and $d_l(d_l^\dagger)$ of the extra particle on an L site chain with periodic boundary conditions. The interaction comes only through the on-site repulsion $U > 0$. If the particle is charged and ϕ depends linearly on time, $\phi = -\mathcal{E}t$, then a constant electric field $\mathcal{E} = -\partial\phi/\partial t$ acts on it. For a vanishing field (it could also be gravitational [2]) the system should follow the adiabatic ground state that we will now map.

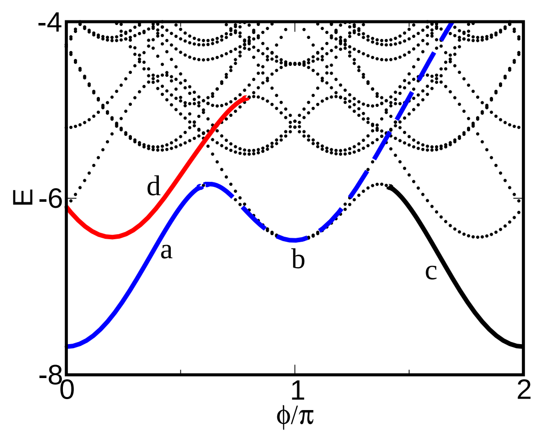

FIG. 1: Low energy spectrum for $L = 10$, $u = 0.5, t_h = t$, along with the reconsruction of selected branches by the Bethe ansatz solution.

For illustration of the typical finite size lattice spectra,

we show in Fig. 1 the evolution of the low lying energy levels for $L = 10$, half- filling and in the $k = 0$ symmetry momentum subspace as a function of ϕ. It is clear that by scanning ϕ we recover at $\phi = 2\pi n/L$ $(n = 0, L-1)$ the spectra in the succesive $k-$subspaces with $\phi = 0$. We will now show how to reconstruct selected branches of the dispersion and study their finite size effects using the Bethe ansatz solution.

The Bethe ansatz wavefunctions, in the presence of flux ϕ, are characterized by $M = N+1$ quantum numbers k_j for N fermions plus the one extra particle, given by the following equations (we take $t_h = t$),

$$k_j = \frac{2\pi I_j}{L} + \frac{1}{L}\theta(\sin k_j - \Lambda), \quad j = 1, \ldots, M \tag{1}$$

$$\theta(p) = -2\tan^{-1}(p/u), \quad u = U/4t \tag{2}$$

$$\begin{aligned} K = \sum_{j=1}^{M} k_j \ &= \ \sum_{j=1}^{M} \frac{2\pi I_j}{L} + \frac{2\pi J}{L} + \phi \\ &= \ \sum_{j=1}^{M} k_j^0 + \frac{2\pi J}{L} + \phi. \end{aligned} \tag{3}$$

The total energy and momentum are given by the sum of quasi-energies and momenta,

$$E = \sum_{j=1}^{M} \epsilon(k_j) = -2t\sum_{j=1}^{M}\cos k_j, \quad K = \sum_{j=1}^{M} k_j. \tag{4}$$

Every state is characterized by a set of half-odd integers I_j and the (half-odd) integer J for (even) odd number of fermions. We will consider the case of odd number of fermions, M even, that is also equivalent to that of hard core bosons in a one dimensional system.

We can reconstruct the low energy states by considering the following branches;
(a) place the $I_j's$ at the successive values $-M/2+1/2 \leq I_j \leq +M/2-1/2$ and scan Λ between $\pm\infty$. At this configuration, for $u \to 0$ and $\Lambda = 0$ the phase shift term in eq.(1) takes the value $+\pi/L(-\pi/L)$ for $k_j < 0 (k_j > 0)$. Thus for one particle $k_j = 0$ while the rest of fermions fill uniformly a Fermi sea, between $-(N-1)/2 \leq k_j \leq +(N-1)/2$. The k-occupation corresponds to that of independent particles.

For a finite u, by varying Λ every k_j shifts by $-\pi/L(+\pi/L)$ for $\Lambda \to -\infty(+\infty)$ so that $-2M\pi/L \leq K \leq +2M\pi/L$. In the thermodynamic limit $-k_F \leq K \leq +k_F$ with $k_F = \pi N/L$. It is clear that the spectrum is scanned by finding the Λ solutions of eqs.(1) as a function of ϕ for $J = 0$ or equivalently for $\phi = 0$ as a function of the quantum number J.
(b) shift each I_j of branch (a) by $+1$. Now by varying $-\infty < \Lambda < +\infty$, $\pi M/L \leq K \leq 3\pi M/L$ which tends in the thermodynamic limit to $k_F \leq K \leq 3k_F$.
(c) shift any I_j of branch (a) by L and scan Λ.
(d) shift I_M of branch (a) by $+1$ and scan Λ.

The adiabatically evolving ground state is obtained by the successive shift of $I_j \to I_j + 1$ up to L times when the $k_j's$ are shifted by the trivial phase 2π. Branch (d) corresponds to a state with an "electron-hole" excitation that is not adiabatically evolving from the ground state.

Next, in Fig. 2 we show the finite size dependence of the adiabatically evolving ground state for a series of lattices at half-filling ($k_F = \pi/2$). It is now clear that in the thermodynamic limit the period becomes $2k_F$. In order

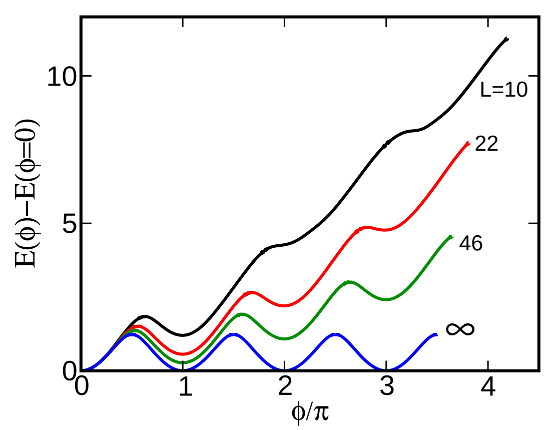

FIG. 2: Finite size scaling of adiabatically evolving ground state for $t_h = t$, $u = 0.5$ and half-filling.

to analyze the finite size effects and obtain the particle dispersion we take to $O(1/L)$,

$$k_j \simeq k_j^0 + \frac{1}{L}\theta(\sin k_j^0 - \Lambda), \quad j = 1, \ldots, M. \tag{5}$$

We can now see how the dispersion becomes symmetric around k_F in the thermodynamic limit by evaluating the difference in energy between the state $K = 0$ and $K = 2k_F$. By shifting $k_1 = (2\pi/L)(-M/2+1/2)$ to $k_{1'} = (2\pi/L)(M/2+1/2)$ and taking $\Lambda = 0$, we obtain,

$$K_{2k_F} - K_0 = \frac{2\pi}{L}M + O(1/L) \simeq 2k_F \tag{6}$$

$$E_{2k_F} - E_0 = +2t\cos(k_1) - 2t\cos(k_{1'}) \simeq \frac{4\pi}{L}\sin k_F. \tag{7}$$

So the finite size effects decrease as $1/L$. Next we can obtain an analytical solution of the dispersion of branch (a) and by symmetry (b) in the thermodynamic limit including corrections $O(1/L)$ from eqs. (3,4,5),

$$\begin{aligned} K \ &= \ \sum_{j=1}^{M} k_j \simeq K_0 + \delta K \\ &= \ \sum_{j=1}^{M} k_j^0 + \frac{1}{L}\sum_{j=1}^{M}\theta(\sin k_j^0 - \Lambda) \end{aligned} \tag{8}$$

$$
\begin{aligned}
E &= -2t\sum_{j=1}^{M}\cos k_j \simeq E_0+\delta E \\
&= -2t\sum_{j=1}^{M}\cos k_j^0+\frac{2t}{L}\sum_{j=1}^{M}\sin k_j^0\theta(\sin k_j^0-\Lambda) \quad (9)
\end{aligned}
$$

Replacing sums by integrals for the branch (a) we find for the momentum and energy dependendent terms on Λ and thus ϕ,

$$
\begin{aligned}
\delta K &= \frac{1}{2\pi}\int_{-k_F}^{+k_F} dk\theta(\sin k-\Lambda) \\
\delta E &= \frac{2t}{2\pi}\int_{-k_F}^{+k_F} \sin k\theta(\sin k-\Lambda). \quad (10)
\end{aligned}
$$

Scanning $-\infty<\Lambda<+\infty$ we obtain the dispersion shown in Fig. 2 for $L\rightarrow+\infty$.

If we interpret $\delta E, \delta K$ as the dispersion of the correlation energy and momentum we can deduce their temperature dependence by inserting in eqs. (8,9) a Fermi-Dirac thermal distribution f_k^0 for the momenta k_j^0.

$$
\begin{aligned}
\delta K &= \frac{1}{2\pi}\int_{-\pi}^{+\pi} dk f_k^0\theta(\sin k-\Lambda), \\
\delta E &= \frac{2t}{2\pi}\int_{-\pi}^{+\pi} f_k^0\sin k\theta(\sin k-\Lambda), \\
f_k^0 &= \frac{1}{1+e^{(\epsilon(k^0)-\mu)/k_BT}} \quad (11)
\end{aligned}
$$

Now, by electron-hole symmetry we see that at half-filling, $\mu=0$, the thermal distribution factors cancel so that the dispersion becomes temperature independent. This is due to the particular form of scattering phase shifts in this integrable model.

The next issue we want to discuss is the response of the particle to a constant external field $\mathcal{E}$ created by a time dependent flux $\phi=-\mathcal{E}t$. As is evident from Fig. 1 the collective ground state evolves through a maze of excited "electron-hole" states by level crossing. The coherent evolution throught the energy spectrum is protected by the macroscopic number of conservation laws characteristic of an integrable system that suppress level repulsion and also result to Poisson statistics. There is a coherent drag of the Fermi sea by the "heavy" particle. We should note that a similar level crossing trajectory also characterizes the time evolution of every excited state. We can plausibly argue that this is a generic behavior of integrable quantum many-body systems.

To put this evolution into evidence we show in Fig. 3 the long time adiabatic energy evolution of an $L=10$ system as a function of phase $\phi=-\mathcal{E}t$ in units of π for $\mathcal{E}\rightarrow 0$. When the momentum of each particle is displaced by 2π so that $\phi=2\pi M$ we have an identical state (in the example at $\phi/\pi=2M, M=6$). Thus the periodicity of the coherent collective motion becomes macroscopic.

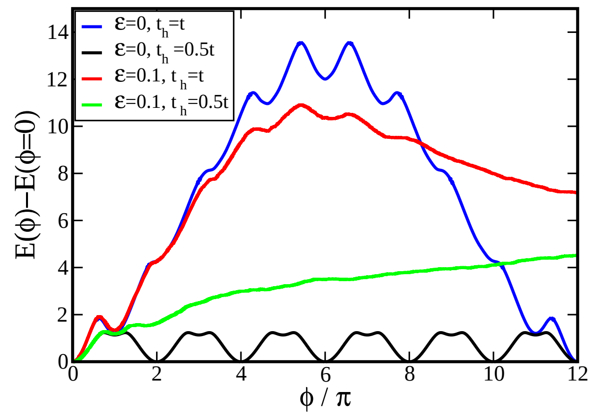

FIG. 3: Time evolution for $L=10$, $u=0.5$ at $t_h=t$, and $t_h=0.5t$.

In the adiabatic limit there is reversible pumping of a macroscopic amount of energy into the system that it could experimentally be observed in a mesoscopic system. In contrast, in the nonintegrable case e.g. $t_h=0.5t$, the periodicity is simply 2π for a finite size system and it becomes $2k_F$ in the thermodynamic limit. This behavior is generic for any interaction U and hopping difference $t_h\neq t$. Note also that in the continuous system, hard core bosons in 1D, the driving leads to an indefinite increase of energy as there is no $\phi=2\pi M$ periodicity.

As shown in Fig. 3, for a finite field e.g. $\mathcal{E}=0.1$, in the integrable case the energy deviates from the adiabatic - nonlinear in energy - trajectory, in sharp contrast to the nonadiabatic case where it spreads diffusively upwards from the ground state. It is clear that in the integrable system the departure from the adiabatic evolution is by mixing with highly excited states while in the nonintegrable case by tunneling between level repelled nearest neighbor states. For long times, the integrable system rapidly evolves through the whole energy spectrum to thermalize at the infinite temperature energy limit (mean value of energy spectrum) in a nonmonotonic way while in the nonintegrable system diffuses monotonically to the same mean energy (not shown). Of course this unconventional thermalization is irrelevant in the thermodynamic limit but it might be observable in a finite size system.

To quantify the spreading of the wavefunction during the evolution, we show in the Fig.4 $\Delta(\phi)$ defined by [10],

$$
\Delta(\phi)=<\Psi(\phi)|(\mathrm{H}(\phi)-<\mathrm{H}(\phi)>)^2|\Psi(\phi)> \quad (12)
$$

Here we also observe that the spreading is highly irregular in the integrable case while it is almost linear in time, diffusive, in the nonintegrable one.

The main issues emerging from these results are, first, whether it is possible to observe experimentally the adi-

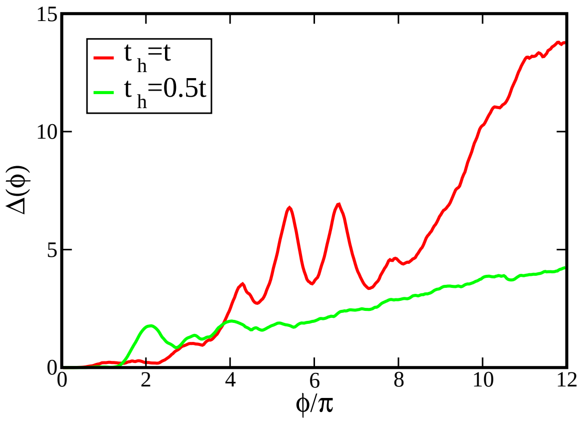

FIG. 4: Time evolution of wavefunction spread $\Delta(\phi)$ for $L = 10$, $u = 0.5$ at $t_h = t$, $t_h = 0.5t$ and $\mathcal{E} = 0.1$.

abatic dispersion of the driven particle in a macroscopic system, that is in fact qualitatively similar in the integrable and nonintegrable case. In this context we should mention that recent works argue on the absence of adiabatic limit in low dimensional gapless systems [11] in macroscopic systems. Still, despite the spreading of the wavefunction, it might still be possible to observe the characteristic energy-momentum dependence for at least a few Bloch oscillations and the remnants of the singular integrable behavior.

Second, in a finite size - mesoscopic - system, there is a striking difference between the nonlinear evolution of an integrable and a nonintegrable system. It is worth studying to what extent this coherent behavior is affected by nonadiabatic effects, for at least some modes of driving. It is experimentally interesting to tune the boson-boson interaction from the weak (nonintegrable) to the strong Tonks-Girardeau (integrable) limit and observe the resulting dynamics. Or alternatively vary the "heavy" particle mass.

We conclude that this prototype integrable model exhibits an unconventional collective nonlinear evolution when driven by a constant field that motivates the exploration of other similar integrable models.

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